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On the Value of Selection as exercised by the Policy-holder against the Company. By JOHN ADAMS HIGHAM, Esq., Fellow, and Member of the Council of the Institute of Actuaries of Great Britain and Ireland, and Actuary to the Royal Exchange Assurance Corporation.

[Read before the Institute of Actuaries, 31st of March, 1851.]

DURING the last session of the Institute of Actuaries, I had the honour to read a paper on the Value of the Principle of Selection as exercised by the Offices in granting Assurances upon Lives, and to submit some calculations which I had made with reference to that subject, in which calculations the lives admitted at particular ages were kept apart from others during their subsequent existence. I have now to offer to the Institute an extended set of Tables of a similar character; and, after stating the course which I have followed in constructing them, I shall make a few observations on the value of selection as exercised by the policy-holder against the Company.

The volume which is now lying on the table, and which will at all times be at the service of the members of the Institute, includes the whole *British* experience published by "the seventeen offices," viz. the "Amicable," the "Equitable," and the fifteen who contributed their data to the Committee of Actuaries of 1838. I judged that, on several accounts, the Irish experience would be unsuited to my purpose, and therefore have not made any use of it.

To lessen the labour, which in any case was likely to be sufficiently great, the lives were combined in quinary classes; those who entered at the ages m, $m+1$, $m+2$, $m+3$ and $m+4$ being regarded as though they had all entered at the mean age (which was always taken a multiple of five). Thus, for example, the table headed "mean age at entry, 40 years," includes all the persons who entered at ages 38, 39, 40, 41 and 42. The following table will show that the numbers thus obtained are, for the most part, large enough to prevent material irregularities in the results.

TABLE A.—*Number of Lives observed.*

Assured at (mean) age		5	82
„	„	10	409
„	„	15	732
„	„	20	2,717
„	„	25	6,593
„	„	30	9,629
„	„	35	10,123
„	„	40	8,606
„	„	45	6,807
„	„	50	4,938
„	„	55	3,209
„	„	60	1,960
„	„	65	924
„	„	70	173
„	„	75	40
			56,942

The materials having thus been classified, it was a very simple matter to form from them columns of the number exposed to risk in each year of age, and the number who survived the year. From the logs. of these columns, the log. of the probability of living a year was obtained, and thence, by the use of Mr. Peter Gray's Tables, the logs. of the curtate expectations were run up with great facility. The annuities were similarly obtained from a column formed by adding log. *v* to the log. of the probability of living a year.

The following is an abstract of the results:—

TABLE B.—*Complete Expectation of Life* (*unadjusted*).

Age.	Assured at Age											
	20.	25.	30.	35.	40.	45.	50.	55.	60.	65.	70.	75.
20	38·29											
25	34·84	36·33										
30	31·25	32·74	32·54									
35	27·93	29·11	28·74	29·95								
40	25·01	25·72	25·21	26·39	27·73							
45	22·07	22·32	21·59	22·89	23·98	23·54						
50	19·42	19·03	18·09	19·40	20·60	19·95	20·34					
55	16·84	16·37	14·94	16·04	17·37	16·77	17·00	17·06				
60	13·77	14·59	12·23	13·07	14·22	13·65	13·87	13·76	14·54			
65	10·43	11·41	10·17	10·34	11·59	10·71	11·16	10·92	11·36	12·13		
70	5·43	7·50	8·60	7·79	9·24	8·40	8·96	8·82	8·97	9·21	10·00	
75	......	2·50	7·14	5·04	8·06	6·70	7·49	6·74	6·89	7·09	8·11	8·75
80	......	......	4·00	3·70	7·10	4·32	6·05	4·83	5·31	5·99	7·39	6·49
85	......	......	......	......	7·09	3·47	4·32	3·20	4·87	4·50	4·99	3·62
90	......	......	......	......	5·38	......	3·44	......	1·67	3·60		

TABLE C.—*Annuities at* 3 *per cent.* (*unadjusted.*)

Age.	Assured at Age											
	20.	25.	30.	35.	40.	45.	50.	55.	60.	65.	70.	75.
20	20·67											
25	19·61	20·30										
30	18·33	19·11	19·13									
35	17·03	17·74	17·66	18·18								
40	15·87	16·36	16·19	16·75	17·26							
45	14·59	14·84	14·46	15·19	15·57	15·47						
50	13·32	13·17	12·61	13·45	13·94	13·69	13·86					
55	12·04	11·80	10·78	11·58	12·22	11·97	12·04	12·16				
60	10·24	11·04	9·10	9·80	10·34	10 12	10·17	10·17	10·69			
65	8·05	9·08	7·75	8·01	8·64	8·19	8·42	8·32	8·63	9·16		
70	4·02	6·23	6·74	6·21	7·00	6·57	6·90	6·90	6·99	7·12	7·64	
75	......	1·91	5·82	4·04	6·17	5·34	5·87	5·38	5·43	5·56	6·31	6·91
80	......	......	3·25	2·96	5·45	3·42	4·81	3·89	4·21	4·73	5·91	5·25
85	......	......	......	......	5·62	2·77	3·44	2·51	3·98	3·59	4·05	2·84
90	......	......	......	......	4·46	......	2·75	......	1·12	2·89		

In considering these tables, it will be borne in mind that they are the results of the rough materials, without any adjustment, or the introduction of any foreign data, except that, in some cases, where persons were left existing at the advanced ages, it was necessary to borrow, from another table, a value for their probable future life-time. It may perhaps be thought that time has been needlessly expended in obtaining these tables from the rough probabilities, seeing that I contemplated an attempt at adjustment. But it was evident that the adjustment must be of so extensive, and in some respects of so arbitrary a character, that it would be necessary, for the satisfaction of the reader, to afford everywhere the opportunity of comparing the adjusted values with the rough values to which they correspond.

In generalising the materials, the first thing to be done was to form a table of probabilities of living a year in respect of mixed lives; in which term I include all the lives, at whatever age assured, among whom, as a body, the effect of selection should appear to have ceased. For example, it may be assumed that persons now aged 75, who were assured at the age of 30, are no better, as a class, than persons, now of the same age, who were assured at the age of 20. As each class would be expected to contain both good and bad in like proportions, they may properly be thrown together in estimating the probability of living a year in respect of mixed lives, and it is necessary that the concluding portions of all the separate tables should be so thrown together, to avoid the irregularities at the older ages occasioned by the small numbers to which the classes have dwindled.

With a view to determine the length of time during which selection continues to exercise an influence, and during which, consequently, the classes must be kept apart, the numbers exposed to risk, and the numbers who survived, were combined for each class in quinary sets, and probabilities of living a year were thence deduced. The following is an example of the resulting table :—

TABLE D.—*Years of risk, ages 70, 71, 72, 73 and 74.*

	Number at risk.	Number who survived.	Probability of living a year.
Lives selected at age 20	4	2	·50000
„ „ 25	4 16	2 16	·50000 1·
„ „ 30	20 286	18 260	·90000 ·90908
„ „ 35	306 978	278 912	·90849 ·93250
„ „ 40	1,284 1,604	1,190 1,470	·92679 ·91647
„ „ 45	2,888 2,126	2,660 1,950	·92104 ·91721
„ „ 50	5,014 2,615	4,610 2,402	·91943 ·91854
„ „ 55	7,629 2,855	7,012 2,650	·91912 ·92820
„ „ 60	10,484 3,390	9,662 3,156	·92159 ·93098
„ „ 65	13,874 3,197	12,818 2,981	·92389 ·93244
„ „ 70	17,071 1,252	15,799 1,171	·92549 ·93532
	18,323	16,970	·92615

Now, assuming that there *is* a virtue in selection, it would be found, but for the irregularities which affect all observations of the kind, that commencing at the bottom of the table and reading upwards, the probabilities would continually diminish until we should reach the class, or the first of the classes, the lives in which have been assured so long that the effect of selection has been exhausted from among them; and from this point still reading upwards, the probabilities would be all alike. The figures in antique type furnish a continuous summation of the separate classes, and it follows from what has just been remarked, that so long as the probability calculated from a particular class (and expressed in common type) exceeds the probability immediately above it (which is derived from all lives now of the same age, but longer insured), so long selection has an effect in that class.

The table on the next page shows, at one view, the whole of these probabilities, both selected and mixed. The line-bar shows the duration of time during which the effect of selection is apparent in the actual probabilities; and as it is rather irregular, the dotted bar shows a modification of its course, which has been made with a view to the adjustment of the resulting tables. This dotted bar has been drawn in conformity with an arbitrary rule, which at the same time was easy of application and in general agreement with the actual figures, viz. that the period during which selection exercises an effect, is half the difference between the age at entry and age 80. At the older ages a rule of this kind will of course fail; and accordingly I assume that after the age of 70 the required period is the same as at that age, viz. 5 years.

TABLE E.

Year of risk.	Age when Assured.														
Age.	5.	10.	15.	20.	25.	30.	35.	40.	45.	50.	55.	60.	65.	70.	75.
7	·99687														
	·99687														
12	·99371	·99408													
	·99371	·99403													
17	1·	·99667	·99170												
	1·	·99697	·99305												
22	1·	·98958	·99181	·99129											
	1·	·99065	·99145	·99129											
27	1·	·99275	·99268	·99120	·99216										
	1·	·99346	·99296	·99154	·99206										
32	1·	1·	·98546	·98937	·99138	·99250									
	1·	1·	·99104	·98971	·99113	·99211									
37	1·	1·	·98394	·98458	·98828	·98917	·99022								
	1·	1·	·99070	·98564	·98787	·98880	·98965								
42		·97958	·98528	·98197	·98664	·98828	·98796	·99074							
		·97958	·98290	·98213	·98594	·98760	·98783	·98919							
47		·94117	1·	·97913	·98360	·98492	·98587	·98562	·98771						
		·94117	·98163	·97947	·98292	·98433	·98521	·98539	·98635						

The Bar marks the period during which the effect of selection is observable, by the excess of the probability in common type over that in antique type, in the preceding column.

—— rough. • • • adjusted.

The subsequent Tables of "Mixed Mortality" are framed by a combination of all the data which supplied the probabilities on the left side of the Bar; the tables of selected or "Class Mortality" are formed from the probabilities on the right of the Bar, and merge in the mixed table at the point where they cross the Bar.

=Prob. of living a Year, determined from those Lives only which were assured at the Age stated at the top of the column.

=Prob. of living a Year, determined from Lives assured at the said Age, and at all [Younger Ages.

52		·93333 ·93333	·92308 ·92856	·97396 ·96819	·97425 ·97333	·97787 ·97663	·98168 ·97942	·98188 ·98057	·98075 ·98064	·98313 ·98152					
57		1· 1·	1· 1·	·97351 ·97789	·96017 ·96256	·96710 ·96603	·97277 ·96988	·97692 ·97315	·97537 ·97400	·97618 ·97474	·97893 ·97598				
62		1· 1·	1· 1·	·96552 ·97500	·97246 ·97279	·94980 ·95475	·96263 ·95956	·96452 ·96201	·96690 ·96385	·96445 ·96405	·96594 ·96458	·97293 ·96663			
67		1· 1·	1· 1·	1· 1·	·97850 ·98247	·92835 ·93882	·94761 ·94443	·95074 ·94779	·94589 ·94704	·94639 ·94683	·94391 ·94604	·95080 ·94717	·96336 ·95028		
72				·5 ·5	1· ·9	·90908 ·90849	·93250 ·92679	·91647 ·92104	·91721 ·91943	·91854 ·91912	·92820 ·92159	·93098 ·92389	·93244 ·92549	·93532 ·92615	
77					·66667 ·66667	·93333 ·90909	·85658 ·86750	·89964 ·88799	·91016 ·89890	·90109 ·89975	·90022 ·89991	·89565 ·89894	·89064 ·89726	·89613 ·89720	·93940 ·89871
82						·87500 ·87500	·83333 ·85000	·85937 ·85714	·91296 ·82960	·88277 ·85532	·85041 ·85361	·83595 ·84965	·87173 ·85332	·91971 ·85747	·92308 ·85973
87						0· 0·	0· 0·	·85714 ·66667	·84000 ·79411	·81679 ·80906	·78219 ·80000	·89285 ·82524	·82075 ·82433	·92856 ·82968	·80952 ·82893
92								1· 1·	No exp. 1·	·80000 ·85714	0· ·80000	·63636 ·73077	·85714 ·75758	No experience. ·75758	 ·75758

In the table just given, the probabilities are represented for the sake of convenience as corresponding to the mean age of the five over which the combination extends. This would be correct if the numbers exposed to risk in each year separately were uniform, and if the probabilities decreased by a law of first differences. But while the probabilities obviously diminish by increasing differences, this is in some cases counterbalanced, and in others much more than counterbalanced by the extreme rapidity with which the numbers at risk diminish. Accordingly the probability derived from the quinary combination has been assumed correct for the mean age in the separate classes, where the probabilities diminish rapidly; and for an age a quarter of a year younger in the case of the mixed lives, where (except at the advanced ages) the second differences are trifling*. At the beginning and end of the separate tables, that is to say for the first five years after selection, and for all ages after 75, this combination has not been made, but the probabilities have been taken separately for every year.

The probability of surviving the year immediately after selection, is a quantity which we have not at present the means of measuring. The approximate value which I have used is combined of the elements exhibited in the following tables, viz.—

First. The probability of surviving to the end of the *calendar* year of admission. The office age at entry being m years, these lives are assumed on an average to have been at risk during the last half of the year following their completion of age $m-1$†.

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* The number exposed to risk at age m is (in the greater part of the table), to the number exposed to risk at age $m+4$, about as 10 to 6. Supposing the intermediate numbers to be 9, 8 and 7, and the probabilities to diminish by second differences, the probability deduced from the quinary combination, which I refer to age $m+1\frac{3}{4}$ as respects the mixed lives, will be

$$= \text{first term} + \frac{7}{4}d_1 + \frac{52}{32}d_2.$$

The real probability is $= \text{first term} + \frac{7}{4}d_1 + \frac{21}{32}d_2.$

The difference is $=\frac{31}{32}d_2$, a quantity so small during the early and middle ages as to be immaterial. As d_2 is negative, the error lies on the side of underrating the chance of life. For this, abundant compensation will be suggested hereafter.

† It is assumed (agreeably to the precedent of the Committee who collected the data) that the assured entered on an average at the middle of the calendar year; being on an average half a year younger than their office age. The Equitable and Amicable data are given for actual calendar years, but they do not afford the means of determining the probability of living from the time of admission to the 1st of January following.

In this table the deaths are doubled, and the lives are supposed to have been at risk during a whole year. (In *all* the other tables both the living and dying are doubled, in order to avoid fractions.)

TABLE F.

Age.	Actuaries' Male Lives.			Actuaries' Female Lives.		
	No. at risk.	No. who survived.	Probability of living a year.	No. at risk.	No. who survived.	Probability of living a year.
19	896	892	·99563	259	255	·98446
24	3013	3005	·99724	231	231	1·
29	4790	4762	·99426	315	311	·98742
34	5261	5233	·99449	384	380	·98969
39	4397	4373	·99449	432	424	·98152
44	3504	3486	·99495	387	387	1·
49	2465	2441	·99038	418	414	·99038
54	1550	1530	·98719	325	325	1·
59	918	900	·98039	246	246	1·
64	436	432	·99083	130	128	·98469
69	93	89	·95697	42	36	·85724
74	28	26	·92854	8	8	1·

Secondly. The probability of surviving the first entire calendar year after entry, the year being on an average that immediately after completion of age *m*, and extending from the end of the sixth to the end of the eighteenth month after entry.

TABLE G.

Age.	Actuaries' Male Lives.			Actuaries' Female Lives.			Equitable.			Amicable.		
	No. at risk.	No. who survived.	Probability of living a year.	No. at risk.	No. who survived.	Probability of living a year.	No. at risk.	No. who survived.	Probability of living a year.	No. at risk.	No. who survived.	Probability of living a year.
20	1442	1424	·98742	435	423	·97230	2939	2927	·99586	146	145	·99312
25	5197	5145	·98992	396	392	·98992	5915	5897	·99678	690	687	·99586
30	8445	8407	·99563	549	539	·98175	7679	7659	·99747	1264	1254	·99197
35	9311	9235	·99197	664	660	·99380	7361	7339	·99678	1463	1449	·99038
40	7716	7660	·99266	761	757	·99472	6085	6061	·99609	1351	1340	·99197
45	6217	6139	·98742	674	664	·98514	4710	4684	·99449	1052	1043	·99129
50	4366	4314	·98810	736	728	·98901	3271	3233	·98855	778	771	·99106
55	2681	2635	·98288	577	565	·97904	2077	2051	·98742	566	552	·97521
60	1579	1525	·96561	404	394	·97521	1186	1172	·98810	371	363	·97836
65	768	730	·95039	232	222	·95697	500	494	·98787	200	195	·97499
70	152	144	·94755	64	64	1·	6	6	1·	69	63	·91306
75	48	48	1·	11	11	1·	8	8	1·	none		-

TABLE H.—*Probability of living a year, derived from a combination of all the foregoing materials.*

Age.	Number at risk.	Number who survived.	Probability of living a year.	Same slightly adjusted.
20	6,117	6,066	·99175	
25	15,442	15,357	·99449	·99586
30	23,042	22,932	·99518	·99518
35	24,444	24,296	·99380	·99426
40	20,742	20,615	·99380	·99380
45	16,544	16,403	·99152	·99152
50	12,034	11,901	·98878	·98878
55	7,776	7,658	·98514	·98514
60	4,704	4,600	·97791	·98017
65	2,266	2,201	·97118	·97118
70	426	402	·94363	·95724
75	103	101	(·98062)	·93975

The general table, in which all the separate classes are merged at the advanced ages, may be considered more accurate for those ages than any at present in use.

It is rather curious to trace the history of the value of 4·75 years' purchase which is so commonly given as the complete expectation of life at age 80. The Committee of 1838 borrowed it from Mr. Morgan, probably because they mistrusted the larger value (5·13) which their own data would have given. Mr. Morgan did not obtain it from his materials. They give a value of 5·51, which, perhaps, he rejected for the same reason. He evidently borrowed from the Northampton Table by Dr. Price, and considering that Dr. Price had no more minute materials than the number of deaths between 80 and 90, and the number between 90 and 100, we may be allowed to question the accuracy of his deductions.

The Carlisle Table, through its whole extent, is of no authority, and at the extreme ages it is absolutely useless and absurd. The numbers on which it is founded are so small, that it owes its popularity entirely to the accident that it happens to agree pretty nearly with other observations.

The values at the older ages in Mr. Davies's Tables are based on a mere assumption, though a happy one; and one cannot receive with entire confidence those of Mr. Farr, considering the loose manner in which the ages of individuals, supposed to be octogenarians, or even older, are returned to the registrars of deaths.

It appears therefore desirable to furnish at length the following Table of Probabilities, both rough and adjusted, for ages 75 and upwards. It will be borne in mind that the actual numbers are but half those stated in the table; still they are large enough to entitle the resulting values to confidence. The lives insured at the age of 75 are not included for the first five years after entry, as the table represents the probabilities of life in respect of persons among whom the effect of selection has been exhausted.

TABLE I.—*Mixed Mortality. Age 75 and upwards.*

Age.	Number at risk.	Number who survived.	Rough probability of living a year.	Rough complete expectation.	Adjusted probability of living a year.	Adjusted complete expectation.
75	2089	1896	·90761	7·06	·90890	7·08
76	1715	1542	·89912	6·72	·90237	6·74
77	1397	1239	·88691	6·42	·89538	6·42
78	1105	993	·89865	6·18	·88805	6·11
79	883	780	·88334	5·82	·88052	5·82
80	720	613	·85139	5·52	·87290	5·54
81	553	495	·89512	5·40	·86533	5·27
82	436	364	·83485	4·98	·85793	5·02
83	326	278	·85276	4·86	·85082	4·76
84	239	205	·85774	4·61	·84414	4·51
85	191	172	·90052	4·30	·83801	4·25
86	154	127	·82467	3·72	·83415	3·98
87	112	85	·75892	3·40	·82432	3·67
88	67	55	·82090	3·32	·80853	3·34
89	43	31	·72093	2·93	·78677	3·02
90	26	20	·76923	2·87	·75904	2·70
91	20	16	·80000	2·59	·72534	2·40
92	12	8	·66667	2·11	·68568	2·12
93	8	6	·75000	1·91	·64005	1·86
94	none	....	....	1·38	·58845	1·63
95	....	....	....		·53088	1·42
96	....	....	....	from	·46734	1·23
97	....	....	....	Davies.	·39783	1·06
98	....	....	....	....	·32235	·91
99	....	....	....	....	·24090	·78
100	....	....	....	....	·15348	·66
101	....	....	....	....	·06009	·56
102	....	....	....	....	0	·50

Comparative Statement. Expectation of Life.

Age.	Rough values from above.	Adjusted.	Actuaries' rough materials.	Morgan's rough materials.	Carlisle.	Davies.	Farr (Males).
75	7·06	7·08	6·67	7·02	7·01	7·52	6·55
80	5·52	5·54	5·13	5·52	5·51	5·38	4·94
85	4·30	4·25	3·99	4·03	4·12	3·73	3·68
90	2·87	2·70	3·14	2·48	3·28	2·65	2·73
95	....	1·42	1·17	....	3·53	1·05	2·02

The probabilities for the years immediately after admission, and those for the old ages having been determined in the manner stated, the middle part of the tables was supplied as follows, viz. the several probabilities immediately on the left-hand of the dotted bar in Table E. having been taken out, the intermediate terms were interpolated, and thus the table for mixed lives hereafter given was obtained.

The values on the right-hand of the bar (after a slight adjustment) were similarly used in computing the tables for selected lives. These tables, of course, merge in the mixed table at the point where they cross the bar, and where, by the rule, the effect of selection ceases in each case.

On comparing the probabilities of living a year in the separate classes with those of Mr. Farr, given in the reports of the Registrar-General, one is struck with the fact that assured lives are, for some time after selection, much better than the community at large, but that after a while they become much worse. If the comparison be made with a table representing the chances of life in the better classes of society, to which assured lives generally belong, their inferiority after a short period of assurance will be still more strongly exhibited. There can be nothing in their constitution or circumstances to account for this; and it can arise from no other cause than *the selection which the assured exercise against the Companies* by dropping policies on healthy lives and retaining those on lives which have become bad or doubtful. Various writers who have noticed the inferiority of assured lives taken in the mass, have hastily concluded that the care which the Companies exercise in the admission of applicants fails of its purpose, through the incompleteness of the information they obtain, and the occasional success of dishonest speculators. But if this were so, it would be apparent from the outset; whereas, in the early years after entry, the reverse is the case. I conclude, therefore, that if we could trace the after-history of *all* who appear upon our books, we should find that they really were, as a whole, the select lives we supposed them to be on their admission;—that those who dropped their policies were those who permanently enjoyed good health; while those who had fallen into bad health were careful to retain them.

I am not aware that attention has previously been called to this point, but it is one deserving of very serious consideration, especially by those who are accustomed to regard lapsed assurances as a source of profit. It might be so if the policies to be dropped were settled by lot; but seeing that a power of selection is exercised in opposition to the interests of the Company, there is every reason to fear that a loss instead of a gain is the result. It is true, a measure of profit is shown upon the immediate transaction, inasmuch as the Company are enabled to dispose of their reserve in respect of the policies surrendered. But this is so obviously done at the expense of the future, that one can hardly hear a Company congratulating themselves on their large profits from discontinued policies, without being reminded of the man in the fable, who cut open his goose to secure the golden eggs.

Assuming the correctness of the Table of Mixed Mortality now submitted, it is evident that the RESERVE of an Assurance Company ought not to be made on the assumption that the duration of the lives assured will be in conformity with the general law; much less should tables showing a *greater* expectation of life be used, such for instance as the Carlisle, unless with a considerable per-centage of addition to the results. This applies with peculiar force to the valuation of the liabilities of a Company with a view to their transfer to another office; because such arrangements must always, more or less, unsettle the assured, and lead to the discontinuance of policies which would otherwise have been kept in force.

The easy terms on which unsuccessful Companies have succeeded in transferring their risks have often occasioned surprise, and have had a mischievous effect in encouraging the formation of speculative offices. One constantly reads also in the reports of Companies which, though respectable, have been so little successful that they ought to be immediately wound up, that the Directors are encouraged to proceed, by knowing that offices can be found who will gladly relieve them of their liabilities, either without consideration, or with one of a very small amount. Such arrangements can only involve loss to the Company, as will appear, I think, from the Comparative Table (O.) given at the end of this paper.

These remarks apply chiefly to lives at the middle ages, which ordinarily constitute the bulk of a Company's business. At the old ages the expectation of life is greater than that by Mr. Farr's Tables. A policy on an old life is rarely surrendered. Be the life never so healthy, the holder keeps it up in the expectation that in the course of nature it must ere long become a claim. The natural state of things is thus restored: the easy circumstances and command of appliances enjoyed by the class of persons whose lives are usually insured, result in a greater longevity at the old ages than that observed in the nation generally, including of course the destitute portion of our population.

Up to this point it has been assumed that the tables represent accurately the mortality which a Company will have to meet; but we have now to notice that, by reason of the imperfection of the data, they certainly *under-rate it.*

With the materials at present available, one can do no more than follow the policies as long as their history is given to us. But here is the imperfection which marks all the tables, founded on office-experience, which we at present possess—they treat all the policies which were existing at the period of observation as though they had lapsed. This would be of no consequence if they were of the same quality as the rest; but if (as I think is clear) they are of the *worst* quality—worse than the lapsed policies—worse than the community at large—it is evident that a Table of Expectations founded on the *complete* experience of Assurance Companies would show values very much below those here given. The measure of effect which would result from tracing all the policies to their termination may be judged from the large proportion which the existing policies bear to the whole number under observation.

TABLE K.

	Actuaries'.	Equitable.	Amicable.	Total.
Number left existing ..	20,706	6,930	2,617	30,253
Number discontinued..	8,186	9,323	606	18,115
Number who died	2,482	5,144	948	8,574
	31,374	21,397	4,171	56,942

This is quite sufficient to explain the heavy mortality which, it is understood, some of the older Companies have sustained. Comparing their lengthened experience with the limited experience published by the Committee of 1838 (limited in point of duration), they have been led to apprehend that they had been unfortunate in the selection of their lives; whereas it is probable that the lives, from whose early history the tables of the Committee were framed, would be found to be no better than the others, if the continued policies were followed to the termination of their existence. The report of the Committee does not trace a single policy beyond the twenty-third year of its existence; and very few extend to anything like that term. Were it not helped out by the Equitable and Amicable experience, the Tables now before the Institute must have stopped short at the twenty-third year; so that, for example, the probabilities of life for persons assured at age 20 could not have been shown beyond age 43, and so on. On this account, any new collection of the experience of Assurance Companies, after the manner of the former one, would be comparatively useless, as it could only afford information on that portion of the inquiry which the Committee have amply investigated.

It is much to be hoped that the Equitable Society may be induced to continue the publication of its experience for the twenty years which have elapsed since its former report. Such a record would be of great value; yet neither it, nor the present Equitable Tables, must be hastily assumed as setting out the rate of mortality which other offices are likely to experience. The peculiar system of dividing the surplus which is followed by the Equitable Society, affords to the older policies such large expectation of Bonus, that there is every inducement to maintain its policies in force; and probably it has suffered much less than other offices from the withdrawal of healthy lives. Possibly it might be found that Bonus offices have in general sustained a lower rate of mortality than non-participating offices—a consideration which should lead to greater caution than is commonly exercised in the calculation of non-participating premiums.

The near agreement of the expectation of life, computed from the aggregate experience of the fifteen offices, with that published by the Equitable Society, is entirely owing to the temporary character of most of the returns made by the former. The great inferiority of the one experience to the other, when fairly compared, will be seen from the following table.

TABLE L.—*Number out of which one died in the calendar year (Jan. to Dec.) next after admission.*

Age when Assured.	Actuaries', Male.	Actuaries', Female.	Equitable.	Amicable.
	One in	One in	One in	One in
20	80	36	242	145
25	99	99	311	242
30	229	55	395	125
35	125	161	311	104
40	136	189	256	125
45	80	67	182	115
50	84	91	87	112
55	58	48	80	40
60	29	40	84	46
65	20	23	82	40

The mortality of the first year after admission has been enormously greater in the case of the fifteen offices than in that of the Equitable. Yet the expectations of life calculated from the whole experience in each case are nearly identical. Nothing can show more clearly the excessive error which must result from supposing that the mortality which an office will experience is truly represented by observations extending on an average over less than five years and a half, and ceasing before the disturbing influence comes fully into operation, to which it is the chief object of this paper to direct attention.

The practice of lending upon policies to the extent of their values—a recent practice as regards most offices—augments the power of selection which the assured are able to exert against the Companies. Formerly a man compelled by embarrassments to raise money upon his policy could only sell it, whatever the health of the life assured. *Now* he sells it if the life be healthy, and borrows the value if the life be unhealthy. I would not be understood to object to lending upon policies; this practice, and that of purchasing them*, are advisable under proper limitations; the true interest of a Company being to afford its customers every reasonable accommodation. It could however be wished that those members of the profession under whose advice the assured are promised that at any time they may obtain on surrender of their policies, or borrow upon them 40 or even 50 per cent. of the premium paid, would consider what they are doing in the light in which it is here endeavoured to place the subject.

Bearing in mind—

1. That our information as to policies of more than about ten or twelve years' duration is almost entirely derived from the Equitable Society;
2. That the lives insured in "the fifteen offices" were of a worse quality from the very outset than those in the Equitable;
3. That the difference of bonus-arrangements is likely to have extended this inferiority into after years;
4. That the system of lending upon policies tends to increase the inferiority still further;
5. That even in the Equitable observations nearly one-third of the policies were left existing;

it will appear very certain that if ever a table should be published, tracing to their termination the whole of the policies of which we now possess a very short history, the probabilities thence resulting would be greatly different from those in the tables, on the use of which, in the valuation of liabilities, recent writers have insisted, upon the alleged, but erroneous, ground that they "truly represent the mortality which the office is likely to experience."

If these considerations are important in connexion with the sufficiency of our reserves, they have an equally material bearing upon the question of premiums. It may easily be shown that many scales of premium demanded in the present day are barely sufficient to meet the risks, on the assumption that the lives assured will drop off in conformity with the experience tables. BUT THEY WILL NOT. The office must pass through the successive periods in the course of which *more than half* of the policies issued are supposed in the construction of the tables to have lapsed, while in fact they remained in force. This assumption was unavoidable, because they passed from under the observation of the Committee; but so far from passing from under the observation of the office, they remained upon its books, a deteriorated and burdensome class.

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* Perhaps there is no department of insurance practice which calls more strongly for reconsideration than that of valuing policies for surrender. There can be no doubt that we constantly pay consideration for the discontinuance of insurances, which it would be better for the Company should remain in force. Those Companies, too, which furnish forms of assignment, arm the assured with an additional power against themselves, by facilitating the transfer of policies to third parties whose knowledge of the state of health of the assured may induce them as a speculation to offer higher terms than the official values.

These contingencies can only be covered by an ample margin being kept upon both premiums and reserves; and those Companies which estimate their liabilities at a minimum, should, I submit, consider the propriety of stipulating in all their contracts that the policies shall be kept up for the whole of life. It is difficult to imagine how they can otherwise meet their engagements.

I will conclude this paper by one or two brief suggestions:—

1. It is probable that the large expectation of life shown by Mr. Neison's Friendly Societies' Tables, may result from the immense number of insurances which are discontinued, not as in the case of the insurance companies by the healthy and prudent, but by the unhealthy and reckless, those who have fallen into bad health, bad circumstances or bad habits.

2. A Company ought not to boast of the small amount of its claims in the early years of its establishment, compared with those provided for by any general experience table. The comparison ought to be made with such tables as Mr. Farren's, or those now submitted. Many a board has indirectly complimented itself by means of a commendation of its medical officer, for care in the selection of lives; when the mortality, if measured by a proper test, would have been found excessive.

3. There is much impropriety in the *ad captandum* use of the terms "true and false table," which is made by those who urge that the Carlisle, or English Life, or some similar Table, should be used for the purpose of valuation. All such tables are false for such a purpose; nor is it possible to use a true one. The preference I have elsewhere expressed for Mr. Davies's Table as

a basis for the calculation of premiums, is confirmed by the present extended observations. It agrees very nearly throughout with the values of lives at the time of selection which those observations furnish. But if it agreed exactly, it would be a false table for every other purpose than that of determining the premium. All subsequent calculations affecting the policy ought to be made from classified tables such as those appended to this paper; not those particular tables, which are but rude approximations to what they should be, but tables in their form. At the moment of entry, Mr. Davies's Table represents (I think) most correctly the value of life for an office whose business is of the best class; after a considerable period of assurance, the Northampton values will be most correct. Under ordinary circumstances, the business to be valued will include policies of all ages; and the best course an actuary can pursue is probably to use a table founded on an average rate of mortality, and to make such a per-centage of addition to the results as may seem to him sufficient to cover the loss arising from the withdrawal of healthy lives. But if, without taking such a margin, he determine his reserve on the assumption that lives long assured will further live as long as other people, and if he aggravate this error by converting his supposed surplus into reversionary bonus on the same assumption, the issue must be ruinous to the Company for which he acts.

JOHN A. HIGHAM.

Royal Exchange Life Office,
March 13, 1851.

TABLE M.—*Mixed Mortality adjusted. Lives from among whom the effect of selection has been exhausted.*

Age.	Probability of living a year.	Complete expectation of life.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation of life.	Annuity, 3 per cent.	Annuity, 3½ per cent.
7	·99669	48·12	23·36	21·27	55	·97092	16·47	11·70	11·18
8	·99600	47·27	23·14	21·10	56	·97039	15·95	11·42	10·91
9	·99538	46·46	22·93	20·91	57	·97002	15·42	11·12	10·64
10	·99482	45·67	22·73	20·75	58	·96973	14·88	10·80	10·35
11	·99434	44·91	22·53	20·59	59	·96911	14·33	10·47	10·04
12	·99398	44·17	22·35	20·44	60	·96769	13·76	10·13	9·73
13	·99378	43·43	22·15	20·28	61	·96566	13·22	9·78	9·40
14	·99358	42·70	21·96	20·12	62	·96296	12·66	9·43	9·08
15	·99339	41·98	21·76	19·96	63	·95939	12·13	9·09	8·76
16	·99319	41·26	21·56	19·80	64	·95586	11·63	8·76	8·45
17	·99299	40·54	21·36	19·63	65	·95233	11·14	8·44	8·15
18	·99274	39·82	21·15	19·46	66	·94876	10·67	8·13	7·85
19	·99252	39·12	20·95	19·29	67	·94515	10·22	7·82	7·57
20	·99231	38·41	20·74	19·12	68	·94153	9·79	7·52	7·29
21	·99215	37·71	20·54	18·94	69	·93764	9·36	7·23	7·01
22	·99208	37·01	20·32	18·76	70	·93347	8·96	6·94	6·74
23	·99201	36·30	20·09	18·58	71	·92901	8·56	6·66	6·47
24	·99196	35·60	19·87	18·38	72	·92445	8·17	6·39	6·21
25	·99185	34·88	19·63	18·18	73	·92007	7·80	6·12	5·95
26	·99169	34·16	19·38	17·97	74	·91484	7·43	5·85	5·69
27	·99149	33·44	19·14	17·75	75	·90890	7·08	5·58	5·44
28	·99128	32·72	18·88	17·53	76	·90237	6·74	5·33	5·20
29	·99098	32·00	18·62	17·30	77	·89538	6·42	5·08	4·96
30	·99059	31·28	18·35	17·07	78	·88805	6·11	4·84	4·73
31	·99012	30·57	18·08	16·84	79	·88052	5·82	4·62	4·52
32	·98950	29·88	17·81	16·60	80	·87290	5·54	4·40	4·31
33	·98865	29·19	17·54	16·36	81	·86533	5·27	4·19	4·11
34	·98782	28·52	17·27	16·14	82	·85793	5·02	3·99	3·91
35	·98701	27·86	17·01	15·90	83	·85082	4·76	3·79	3·72
36	·98623	27·23	16·75	15·67	84	·84414	4·51	3·59	3·52
37	·98545	26·60	16·49	15·44	85	·83801	4·25	3·38	3·32
38	·98470	25·99	16·24	15·23	86	·83415	3·98	3·15	3·10
39	·98397	25·38	15·98	15·00	87	·82432	3·67	2·89	2·85
40	·98328	24·79	15·73	14·78	88	·80853	3·34	2·61	2·58
41	·98262	24·20	15·47	14·55	89	·78677	3·02	2·33	2·30
42	·98207	23·62	15·22	14·33	90	·75904	2·70	2·05	2·02
43	·98178	23·05	14·97	14·10	91	·72534	2·40	1·78	1·76
44	·98136	22·47	14·70	13·88	92	·68568	2·12	1·52	1·51
45	·98079	21·89	14·43	13·63	93	·64005	1·86	1·29	1·28
46	·98008	21·31	14·15	13·39	94	·58845	1·63	1·07	1·06
47	·97911	20·72	13·87	13·13	95	·53088	1·42	·88	·87
48	·97768	20·16	13·60	12·88	96	·46734	1·23	·70	·69
49	·97637	19·61	13·33	12·64	97	·39783	1·06	·54	·54
50	·97517	19·07	13·06	12·40	98	·32235	·91	·40	·40
51	·97408	18·54	12·79	12·15	99	·24090	·78	·27	·27
52	·97310	18·02	12·52	11·92	100	·15348	·66	·16	·16
53	·97224	17·50	12·26	11·68	101	·06009	·56	·06	·06
54	·97153	16·99	11·99	11·43	102	....	·50		

TABLE N.—*Class Mortality adjusted.*

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
Lives assured at age 25.									
25	·99586	36·50	20·32	18·78	40	·98749	25·86	16·35	15·36
26	·99220	35·66	20·01	18·52	41	·98723	25·18	16·05	15·09
27	·99194	34·93	19·77	18·32	42	·98696	24·51	15·75	14·83
28	·99167	34·21	19·53	18·12	43	·98680	23·82	15·43	14·55
29	·99140	33·49	19·28	17·91	44	·98640	23·13	15·11	14·26
30	·99113	32·78	19·04	17·70	45	·98577	22·45	14·78	13·96
31	·99086	32·07	18·78	17·48	46	·98490	21·75	14·44	13·66
32	·99059	31·37	18·53	17·26	47	·98380	21·08	14·11	13·35
33	·99013	30·66	18·27	17·03	48	·98188	20·42	13·77	13·05
34	·98967	29·95	18·00	16·80	49	·97995	19·79	13·44	12·75
35	·98921	29·27	17·74	16·58	50	·97802	19·17	13·13	12·47
36	·98875	28·58	17·47	16·34	51	·97610	18·59	12·83	12·20
37	·98828	27·89	17·19	16·10	52	·97417	18·04	12·53	11·93
38	·98802	27·22	16·92	15·86	53	·97224	17·50	12·25	11·68
39	·98776	26·54	16·64	15·61	Merged from hence in Mixed Table.				
Lives assured at age 30.									
30	·99518	33·48	19·37	17·99	44	·98675	23·55	15·36	14·48
31	·99129	32·64	19·05	17·71	45	·98621	22·86	15·03	14·19
32	·99094	31·93	18·80	17·49	46	·98567	22·17	14·70	13·89
33	·99059	31·22	18·54	17·27	47	·98512	21·48	14·36	13·59
34	·99023	30·51	18·28	17·04	48	·98385	20·80	14·02	13·28
35	·98988	29·81	18·01	16·82	49	·98257	20·13	13·68	12·97
36	·98953	29·11	17·74	16·58	50	·98130	19·48	13·34	12·66
37	·98917	28·41	17·47	16·35	51	·98003	18·84	13·00	12·35
38	·98891	27·71	17·19	16·10	52	·97875	18·22	12·66	12·05
39	·98864	27·02	16·89	15·85	53	·97614	17·60	12·33	11·74
40	·98837	26·32	16·61	15·59	54	·97353	17·02	12·00	11·45
41	·98810	25·63	16·31	15·32	55	·97092	16·47	11·70	11·18
42	·98783	24·93	16·00	15·05	Merged from hence in Mixed Table.				
43	·98729	24·24	15·69	14·77					
Lives assured at age 35.									
35	·99426	30·38	18·30	17·06	48	·98458	21·17	14·26	13·50
36	·99015	29·56	17·96	16·76	49	·98368	20·49	13·91	13·19
37	·98981	28·85	17·68	16·52	50	·98278	19·82	13·56	12·87
38	·98947	28·14	17·39	16·28	51	·98189	19·16	13·21	12·55
39	·98912	27·43	17·11	16·03	52	·98099	18·50	12·86	12·23
40	·98878	26·73	16·82	15·77	53	·97912	17·85	12·50	11·91
41	·98844	26·03	16·52	15·51	54	·97724	17·22	12·15	11·60
42	·98809	25·33	16·21	15·24	55	·97536	16·62	11·80	11·27
43	·98757	24·63	15·90	14·96	56	·97349	16·02	11·47	10·96
44	·98705	23·93	15·58	14·68	57	·97161	15·45	11·14	10·65
45	·98653	23·24	15·26	14·40	58	·96973	14·88	10·80	10·35
46	·98600	22·54	14·93	14·10	Merged from hence in Mixed Table.				
47	·98547	21·86	14·59	13·80					

TABLE N.—*Class Mortality adjusted* (continued).

Lives assured at age 40.

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
40	·99380	27·19	17·06	16·00	51	·98217	19·44	13·38	12·72
41	·98946	26·36	16·68	15·66	52	·98115	18·79	13·03	12·40
42	·98855	25·64	16·36	15·38	53	·98012	18·14	12·69	12·08
43	·98809	24·93	16·05	15·11	54	·97908	17·49	12·33	11·76
44	·98756	24·22	15·73	14·83	55	·97804	16·86	11·98	11·44
45	·98697	23·52	15·40	14·54	56	·97700	16·22	11·61	11·10
46	·98632	22·83	15·08	14·24	57	·97596	15·59	11·24	10·76
47	·98562	22·14	14·75	13·95	58	·97363	14·96	10·86	10·41
48	·98485	21·45	14·41	13·65	59	·97087	14·35	10·49	10·07
49	·98402	22·77	14·07	13·34	60	·96769	13·76	10·13	9·73
50	·98313	20·10	13·73	13·03	Merged from hence in Mixed Table.				

Lives assured at age 45.

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
45	·99152	23·77	15·56	14·68	55	·97910	16·94	12·04	11·49
46	·98696	22·97	15·17	14·32	56	·97775	16·29	11·67	11·15
47	·98605	22·27	14·83	14·02	57	·97609	15·66	11·29	10·80
48	·98514	21·58	14·49	13·71	58	·97412	15·03	10·91	10·45
49	·98421	20·89	14·15	13·41	59	·97215	14·42	10·54	10·11
50	·98326	20·22	13·80	13·10	60	·96956	13·81	10·16	9·76
51	·98229	19·55	13·46	12·79	61	·96666	13·23	9·80	9·42
52	·98129	18·90	13·12	12·48	62	·96345	12·67	9·44	9·08
53	·98087	18·25	12·77	12·16	63	·95939	12·13	9·09	8·76
54	·98014	17·60	12·41	11·83	Merged from hence in Mixed Table.				

Lives assured at age 50.

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
50	·98878	20·49	13·98	13·26	59	·97248	14·46	10·57	10·14
51	·98378	19·72	13·57	12·89	60	·97005	13·86	10·20	9·79
52	·98265	19·04	13·21	12·56	61	·96737	13·27	9·83	9·45
53	·98193	18·36	12·84	12·23	62	·96445	12·70	9·46	9·11
54	·98097	17·69	12·47	11·89	63	·96041	12·15	9·11	8·77
55	·97976	17·02	12·09	11·54	64	·95637	11·63	8·76	8·45
56	·97831	16·37	11·71	11·19	65	·95233	11·14	8·44	8·15
57	·97661	15·72	11·33	10·84	Merged from hence in Mixed Table.				
58	·97467	15·08	10·95	10·49					

Lives assured at age 55.

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
55	·98514	17·35	12·30	11·74	63	·96188	12·21	9·15	8·81
56	·97972	16·60	11·87	11·34	64	·95781	11·68	8·80	8·49
57	·97836	15·93	11·48	10·98	65	·95374	11·17	8·46	8·17
58	·97684	15·27	11·08	10·61	66	·94967	10·69	8·14	7·86
59	·97484	14·63	10·69	10·25	67	·94560	10·23	7·83	7·57
60	·97235	13·99	10·29	9·88	68	·94153	9·79	7·53	7·29
61	·96939	13·37	9·90	9·52	Merged from hence in Mixed Table.				
62	·96594	12·78	9·52	9·16					

TABLE N.—*Class Mortality adjusted* (continued).

Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.	Age.	Probability of living a year.	Complete expectation.	Annuity, 3 per cent.	Annuity, 3½ per cent.
Lives assured at age 60.									
60	·98017	14·54	10·68	10·24	66	·95456	10·82	8·24	7·96
61	·97387	13·82	10·22	9·82	67	·94960	10·31	7·89	7·63
62	·97230	13·18	9·81	9·43	68	·94443	9·83	7·56	7·32
63	·96818	12·54	9·39	9·04	69	·93905	9·38	7·24	7·02
64	·96385	11·93	8·99	8·67	70	·93347	8·96	6·94	6·74
65	·95931	11·36	8·61	8·30	Merged from hence in Mixed Table.				
Lives assured at age 65.									
65	·97118	11·83	8·97	8·65	70	·93863	9·04	7·02	6·81
66	·96339	11·17	8·51	8·22	71	·93244	8·60	6·70	6·50
67	·95720	10·58	8·10	7·83	72	·92625	8·19	6·40	6·22
68	·95101	10·03	7·71	7·46	73	·92007	7·80	6·12	5·95
69	·94482	9·52	7·35	7·12	Merged from hence in Mixed Table.				
Lives assured at age 70.									
70	·95724	9·47	7·36	7·13	74	·91813	7·46	5·87	5·71
71	·94579	8·87	6·92	6·71	75	·90890	7·08	5·58	5·44
72	·93657	8·35	6·53	6·35	Merged from hence in Mixed Table.				
73	·92735	7·88	6·18	6·01					
Lives assured at age 75.									
75	·93975	7·62	6·03	5·87	79	8·8584	5·85	4·65	4·54
76	·92464	7·08	5·61	5·47	80	8·7290	5·54	4·40	4·31
77	·91171	6·62	5·25	5·12	Merged from hence in Mixed Table.				
78	·89877	6·21	4·92	4·81					

TABLE O.—*Comparative Table. Complete Expectation of Life.*

Age.	Mixed Mortality.	Class Mortality, Lives assured at Age											English Life Table, Males.	Davies's Equitable.	Carlisle.	Northampton.
		25.	30.	35.	40.	45.	50.	55.	60.	65.	70.	75.				
10	45·67	....	....	....	....	....	....	....	....	....	....	....	47·08	48·83	48·82	39·78
15	41·98	....	....	....	....	....	....	....	....	....	....	....	43·35	44·81	45·00	36·51
20	38·41	....	....	....	....	....	....	....	....	....	....	....	39·88	41·06	41·46	33·43
25	34·88	36·50	....	....	....	....	....	....	.....	....	....	....	36·48	37·44	37·86	30·85
30	31·28	32·78	33·48	....	....	....	....	....	....	....	....	....	33·13	33·98	34·34	28·27
35	27·86	29·27	29·81	30·38	....	....	....	....	....	....	....	....	29·84	30·66	31·00	25·68
40	24·79	25·86	26·32	26·73	27·19	....	....	....	....	....	....	....	26·57	27·40	27·61	23·08
45	21·89	22·45	22·86	23·24	23·52	23·77	....	....	....	....	....	....	23·31	24·10	24·46	20·52
50	19·07	19·17	19·48	19·82	20·00	20·22	20·49	....	....	....	....	....	20·03	20·83	21·11	17·99
55	16·47	16·47	16·47	16·62	16·86	16·94	17·02	17·35	....	....	....	....	16·68	17·85	17·58	15·58
60	13·76	....	....	13·76	13·76	13·81	13·86	13·99	14·54	....	....	....	13·59	15·06	14·34	13·21
65	11·14	....	....	....	....	11·14	11·14	11·17	11·36	11·83	....	....	10·87	12·35	11·79	10·88
70	8·96	....	....	....	....	....	....	8·96	8·96	9·04	9·47	....	8·52	9·84	9·18	8·60
75	7·08	....	....	....	....	....	....	....	....	7·08	7·08	7·62	6·55	7·52	7·01	6·54
80	5·54	....	....	....	....	....	....	....	....	....	....	5·54	4·94	5·38	5·51	4·75
85	4·25	....	....	....	....	....	....	....	....	....	....	....	3·68	3·73	4·12	3·37
90	2·70	....	....	....	....	....	....	....	....	....	....	....	2·73	2·65	3·28	2·41
95	1·42	....	....	....	....	....	....	....	....	....	....	....	2·02	1·05	3·53	·75
100	·66	....	....	....	....	....	....	....	....	....	....	....	1·51	....	2·28	